2.1A What is rate of change p. 66 # 1–4, 63, 64

1. $y = 16t^2$

1. $f(3) = 16(3)^2$

$= 16(9)$

144 total ft in 3 sec.

$= 48$ ft/sec

2. $f(4) = 16(4)^2$

$= 16(16)$

$= 256$

total ft in 4 sec.

$= 64$ ft/sec.

3. $f(3) = 144$

$f(h+3) = 16(h+3)^2$

$$\frac{16(h+3)^2 - 144}{h}$$

$$\frac{16(h^2+6h+9) - 144}{h}$$

$$\frac{16h^2 + 96h + \cancel{144} - \cancel{144}}{h}$$

$$\frac{\cancel{h}(16h + 96)}{\cancel{h}}$$

$$\lim_{h \to 0} 16h + 96 = 96 \text{ ft/sec}$$

4. $f(4) = 256$ ft/se

$f(h+4) = 16(h+4)^2$

$= 16(h^2 + 8h + 16)$

$= 16h^2 + 128h + 256$

$$\frac{16h^2 + 128h + \cancel{256} - \cancel{256}}{h}$$

$$\lim_{h \to 0} \frac{16\cancel{h}(\overset{0}{\cancel{h}} + 8)}{\cancel{h}} = 128 \text{ ft/sec}$$

63. $y = 4.9t^2$ m/t sec.

(a) Average Speed @ $t = 3$

$f(3) = 4.9(3)^2$

$f(3) = 44.1$

$t_{AVE} = 14.7$ m/s

$$\lim_{h \to 0} \frac{4.9h^2 + 29.4h + \cancel{44.1} - \cancel{44.1}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(\overset{0}{\cancel{4.9h}} + 29.4)}{\cancel{h}}$$

$t_{ins} = 29.4$ ft/sec

(b) $f(3) = 44.1$

$f(h+3) = 4.9(h+3)^2$

$= 4.9(h^2 + 6h + 9)$

$= 4.9h^2 + 29.4h + 44.1$

64. $y = gt^2 \, m/t$ sec. $g$ is a constant

Find $g$

a.) $20 = g(4)^2$ $\quad y = \frac{5}{4}t^2 \; m/t$ sec.

$\frac{20}{16} = \frac{g(16)}{16}$

$g = \frac{5}{4}$

b.) Average Speed @ $t = 4$

$f(4) = \frac{5}{4}(4)^2$

$= \frac{5}{4}(16)$

$= 20$ m in 4 sec.

5 m/sec.

C. Instantaneous speed

$f(4) = 20m$

$f(4+h) = \frac{5}{4}(h+4)^2$

$= \frac{5}{4}(h^2 + 8h + 16)$

$= \frac{5}{4}h^2 + 10h + 20$

$$\lim_{h \to 0} \frac{\frac{5}{4}h^2 + 10h + \cancel{20} - \cancel{20}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(\cancel{\tfrac{5}{4}h}^{0} + 10)}{\cancel{h}} = 10 \text{ m/sec.}$$

2.1B The limit of a function: a basic definition p. 66 #7–11, 13, 14
*include a sketch

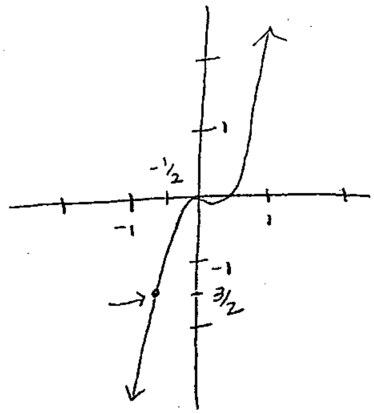
7. $\lim\limits_{x \to -\frac{1}{2}} 3x^2(2x-1)$

$3\left(-\frac{1}{2}\right)^2\left(2\left(-\frac{1}{2}\right)-1\right)$

$3\left(\frac{1}{4}\right)(-1-1)$

$\frac{3}{4}(-2)$

$= -\frac{6}{4}$ or $\boxed{-\frac{3}{2}}$

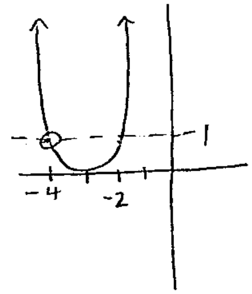



8. $\lim\limits_{x \to -4} (x+3)^{1998}$

$(-4+3)^{1998}$

$= (-1)^{1998}$

$\boxed{=1}$



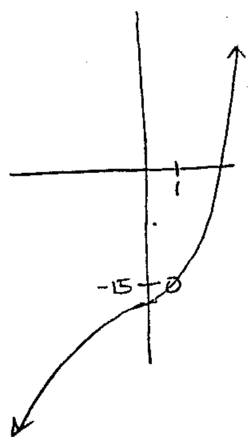

9. $\lim\limits_{x \to 1} x^3+3x^2-2x-17$

$f(1) = (1)^3+3(1)^2-2(1)-17$

$= 1+3-2-17$

$= 4-19$

$\boxed{=-15}$



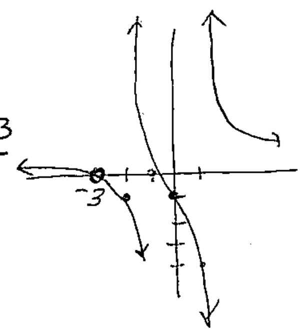

11. $\lim\limits_{y \to -3} \dfrac{y^2+4y+3}{y^2-3}$

$f(-3) = \dfrac{(-3)^2+4(-3)+3}{(-3)^2-3}$

$= \dfrac{9-12+3}{9-3}$

$= \dfrac{-3+3}{6}$

$\boxed{=0}$ $(y+2)(y+3)$



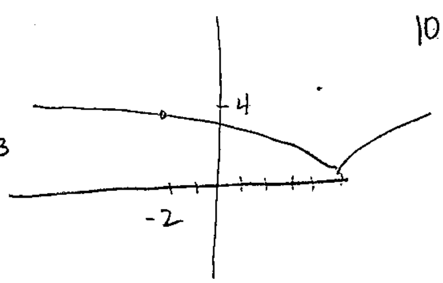

13 ~~14.~~ $\lim\limits_{x \to -2} (x-6)^{2/3}$

$f(-2) = (-2-6)^{2/3}$

$= (-8)^{2/3}$

$= (-2)^2$

$\boxed{=4}$

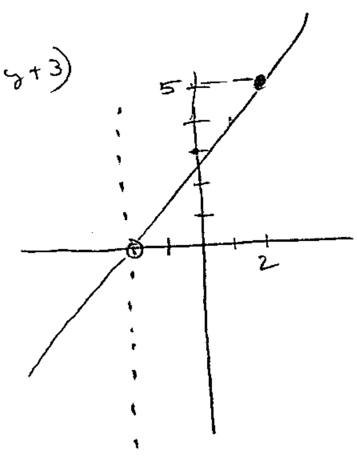



10. $\lim\limits_{y \to 2} \dfrac{y^2+5y+6}{y+2}$

$f(2) = \dfrac{2^2+5(2)+6}{2+2}$

$= \dfrac{4+10+6}{4}$

$= \dfrac{20}{4}$ $\boxed{=5}$



2.1C Finding Limits of Functions: various methods

p. 66 # 19-23; 45-48

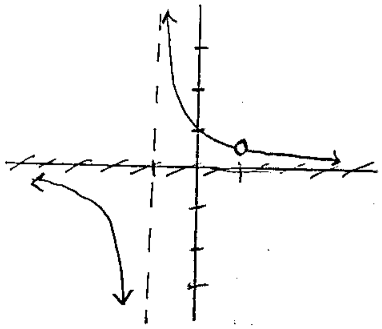
19. $\lim\limits_{x \to 1} \dfrac{x-1}{x^2-1}$

$\dfrac{x-1}{(x+1)(x-1)} = \dfrac{1}{x+1}$

VA @ $x=-1$

hole @ $x=1$

HA @ $y=0$



$\lim\limits_{x \to 1} \dfrac{1}{x+1} = \dfrac{1}{2}$ ✓

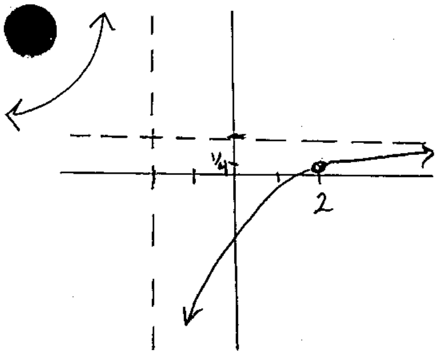
$f(1) = \frac{0}{0} \therefore$ DNE

$f(.9999) \approx .500$

$f(1.0001) \approx .499$

$\boxed{L \approx .5}$ ✓

20. $\lim\limits_{t \to 2} \dfrac{t^2-3t+2}{t^2-4}$




$f(2) = \frac{0}{0} \therefore$ DNE

$f(1.999) \approx .24981$

$f(2.001) \approx .25019$

$\dfrac{\cancel{(t-2)}(t-1)}{(t+2)\cancel{(t-2)}} = \dfrac{t-1}{(t+2)}$

VA @ $t=-2$

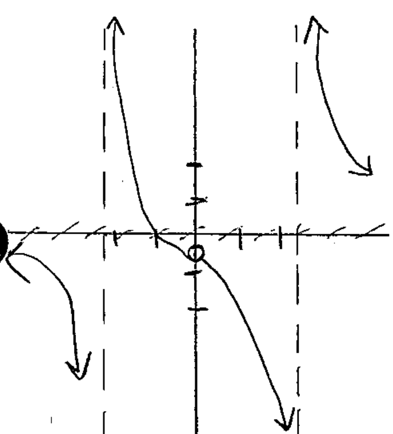
hole @ $t=2$

HA @ $y=1$

$\lim\limits_{t \to 2} \dfrac{t-1}{t+2} = \dfrac{(2-1)}{(2+2)} = \dfrac{1}{4}$ ✓

By Graph $\boxed{L \approx .250}$ ✓

21. $\lim\limits_{x \to 0} \dfrac{5x^3+8x^2}{3x^4-16x^2}$



$f(0) = \frac{0}{0} \therefore$ DNE

$f(.001) \approx -.5003$

$f(-.001) \approx -.4997$

$\boxed{L \approx -.5}$ ✓

$\dfrac{\cancel{x^2}(5x+8)}{\cancel{x^2}(3x^2-16)}$

VA @ $x = \pm\frac{4\sqrt{3}}{3}$

hole @ $x=0$

HA @ $y=0$

$3x^2-16=0$

$\dfrac{3x^2}{3} = \dfrac{16}{3}$

$x \neq \pm\frac{4\sqrt{3}}{3}$

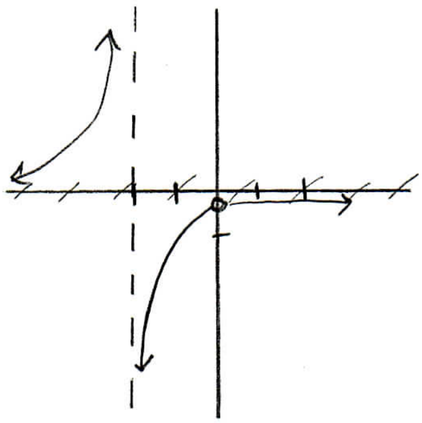
$\lim\limits_{x \to 0} \dfrac{5x+8}{3x^2-16} = \dfrac{-1}{2}$ ✓

22. $\lim_{x \to 0} \dfrac{\frac{1}{2+x} - \frac{1}{2}}{x}$

$f(0) = \frac{0}{0} \therefore$ DNE

$f(0.001) \approx -.2499$ $L \approx -.25$ ✓

$f(-0.001) \approx -.2501$



$\dfrac{\frac{2 \cdot 1}{2 \cdot (2+x)} - \frac{1 \cdot (2+x)}{2 \cdot (2+x)}}{x}$

VA @ $x = -2$

hole @ $x=0$

HA @ $y=0$

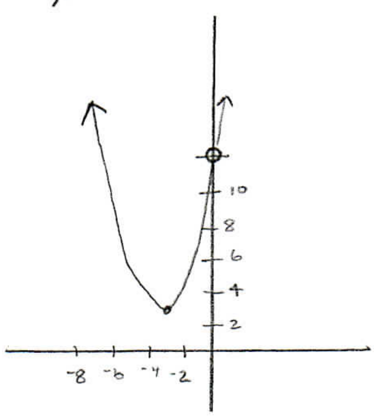
$\dfrac{\cancel{2} - \cancel{2} - x}{2(2+x)} \cdot \dfrac{1}{x} = \dfrac{-1}{2(2+x)}$ $\lim_{x \to 0} \dfrac{-1}{2(2+x)} = -\dfrac{1}{4}$ ✓

23. $\lim_{x \to 0} \dfrac{(2+x)^3 - 8}{x}$

$f(0) = \frac{0}{0} \therefore$ DNE

$f(0.001) \approx 12.006$ $\boxed{L \approx 12.0}$

$f(-0.001) \approx 11.994$




$1(2)^3 \quad 3(2)^2 x \quad 3(2)^1 x^2 \quad 1x^3$

hole @ $x=0$

$\dfrac{\cancel{8} + 12x + 6x^2 + x^3 - \cancel{8}}{x}$

$\dfrac{\cancel{x}(x^2 + 6x + 12)}{\cancel{x}}$ $\lim_{x \to 0} \cancel{x^2} + \cancel{6x} + 12 = 12$

45. $y_1 = \dfrac{x^2 + x - 2}{x - 1}$

$\dfrac{(x+2)(x-1)}{(x-1)}$

hole @ $x = 1$

C

46. $y_1 = \dfrac{x^2 - x - 2}{x - 1}$

$\dfrac{(x-2)(x+1)}{(x-1)}$

VA @ $x = 1$

B

47. $y_1 = \dfrac{x^2 - 2x + 1}{x - 1}$

$\dfrac{(x-1)(x-1)}{(x-1)}$

hole @ $x = 1$

D

48. $y_1 = \dfrac{x^2 + x - 2}{x + 1}$

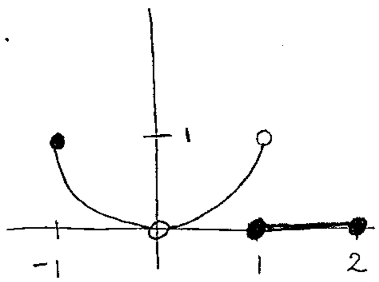
$\dfrac{(x+2)(x-1)}{(x+1)}$

VA @ $x = -1$

A

2.1D Types of Limits: One-sided and Two-Sided p. 66 # 37-40, $\underbrace{44, 51, 52}_{\text{need to finish}}$

37.



a.) $\lim_{x \to -1^+} f(x) = 1$ True

b.) $\lim_{x \to 0^-} f(x) = 0$ True

c.) $\lim_{x \to 0^-} f(x) = 1$ False

d.) $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$ TRue

e.) $\lim_{x \to 0} f(x)$ exists TRue

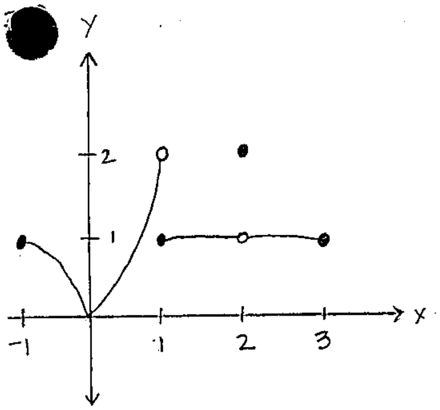
f.) $\lim_{x \to 0} f(x) = 0$ TRue

g.) $\lim_{x \to 0} f(x) = 1$ False

h.) $\lim_{x \to 1} f(x) = 1$ False

i.) $\lim_{x \to 1} f(x) = 0$ False

j.) $\lim_{x \to 2^-} f(x) = 2$ False



a.) $\lim_{x \to -1^+} f(x) = 1$ TRue

b.) $\lim_{x \to 2} f(x)$ does not exist False

c.) $\lim_{x \to 2} f(x) = 2$ False

d.) $\lim_{x \to 1^-} f(x) = 2$ TRue

e.) $\lim_{x \to 1^+} f(x) = 1$ TRue

f.) $\lim_{x \to 1} f(x)$ does not exist TRue

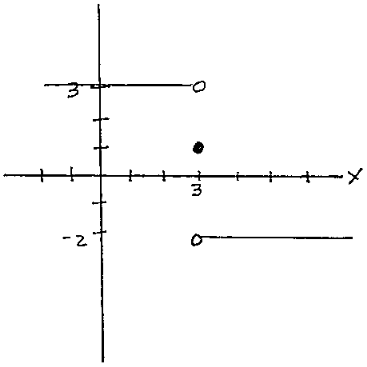
g.) $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$ TRue

h.) $\lim_{x \to c} f(x)$ exists at every c in $\underbrace{(-1, 1)}_{\text{interval?}}$

a limit exists for every x-value between -1 and 1.

i.) $\lim_{x \to c} f(x)$ exists at every c in (1, 3)

39.



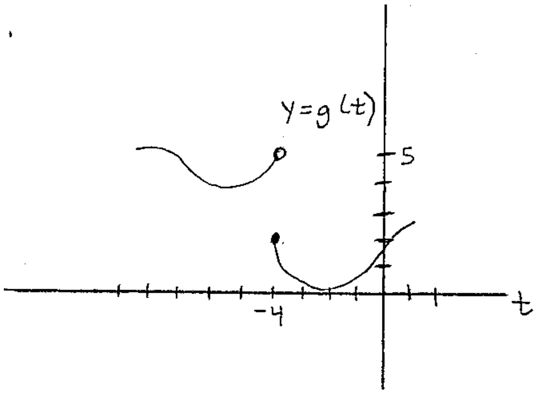

a.) $\lim\limits_{x\to 3^-} f(x) = \boxed{3}$

b.) $\lim\limits_{x\to 3^+} f(x) = \boxed{-2}$

c.) $\lim\limits_{x\to 3} f(x) = \boxed{\text{DNE}}$

d.) $f(3) = \boxed{1}$

40.



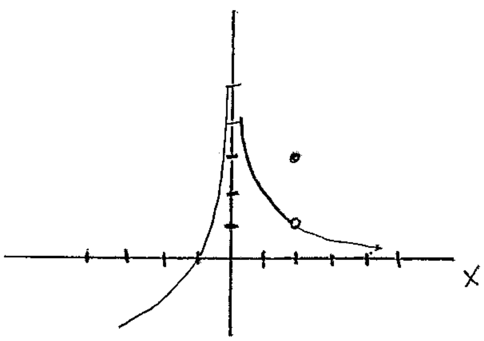

a.) $\lim\limits_{t\to -4^-} g(t) = \boxed{5}$

b.) $\lim\limits_{t\to -4^+} g(t) = \boxed{2}$

c.) $\lim\limits_{t\to -4} g(t) = \boxed{\text{DNE}}$

d.) $g(-4) = \boxed{2}$

44.




a.) $\lim\limits_{x\to 2^-} G(x) = 1$

b.) $\lim\limits_{x\to 2^+} G(x) = 1$

c.) $\lim\limits_{x\to 2} G(x) = 1$

d.) $G(2) = 3$

WHAT IS A LIMIT?

Name:____________________
Period:__________ Date:__________

By completing this activity, you will discover what a limit is, when it exists, and when it doesn't exist.

Let $f(x)=\dfrac{x^2-3x-4}{x+1}$. $\dfrac{(x-4)(x+1)}{x+1} = x-4$

1. What is the domain of f(x)?

   $x \in \mathbb{R}$ but $x \neq -1$

2. What does the graph of f(x) look like? Explain in words without drawing!

   Linear w/ a hole @ $x=-1$

3. The table below gives x-values that are less than but increasingly closer and closer to – 1. These values are said to be approaching -1 from the left. Use your calculator to fill in the missing values of f(x) for each x.

| x | -3 | -2 | -1.5 | -1.25 | -1.1 | -1.001 |
|---|---|---|---|---|---|---|
| f(x) | -7 | -6 | -5.5 | -5.25 | -5.1 | -5.001 |

4. The y-value (or height) you are approaching as you near the x value of – 1 in the table above is called the left-hand limit of -1 and is written $\lim\limits_{x\to -1^-} f(x)$. What is the left-hand limit of f(x)?

   -5

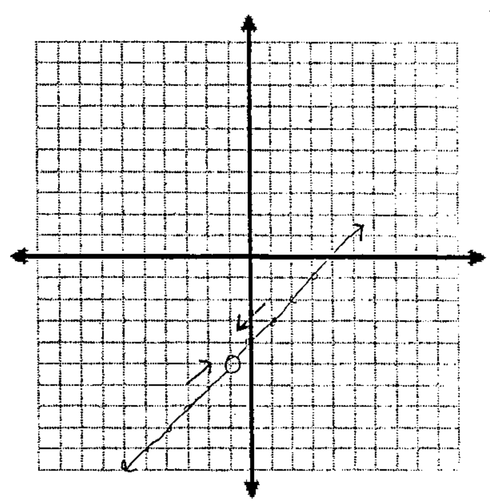
5. The table below gives x-values that are greater than but increasingly closer and closer to – 1. These values are said to be approaching -1 from the right. Use your calculator to fill in the missing values of f(x) for each x.

| x | 1 | 0 | -0.5 | -0.75 | -0.9 | -0.999 |
|---|---|---|---|---|---|---|
| f(x) | -3 | -4 | -4.5 | -4.75 | -4.9 | -4.999 |

6. The y-value (or height) you are approaching as you near the x value of -1 in the table above is called the right-hand limit of -1 and is written $\lim\limits_{x\to -1^+} f(x)$. What is the right-hand limit of f(x)?

   -5

7. Graph f(x), and draw the left- and right-hand limits as arrows on the graph.



8. When the left and right-hand limits as x approaches -1 both exist and are equal, the general limit at $x=-1$ exists and is written $\lim_{x\to -1} f(x)$. Does the general limit exist at $x=-1$? yes If so, what is it?

$-5$ $\lim_{x\to -1} f(x) = -5$

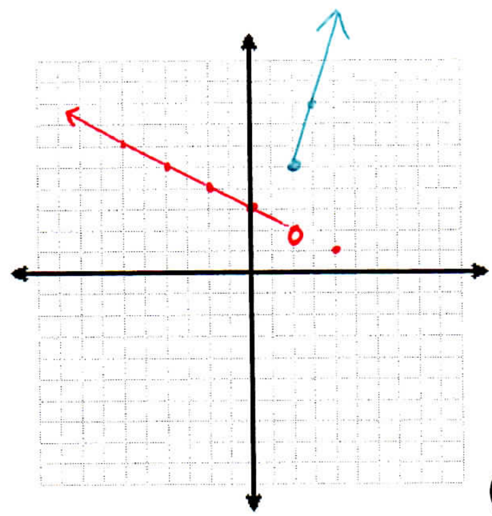
9. Write a few sentences describing what a limit is and how it is found.

A limit represent an output value as the inputs · from the left and right of a given x-value get closer to that x-value. If the outputs equal the same value, the limit exists.

10. Each of the following graphs has no limit at the indicated point. Use a graphing calculator and your knowledge of limits to determine why the limits do not exist. You may need to zoom in or change your window. Draw the graph.

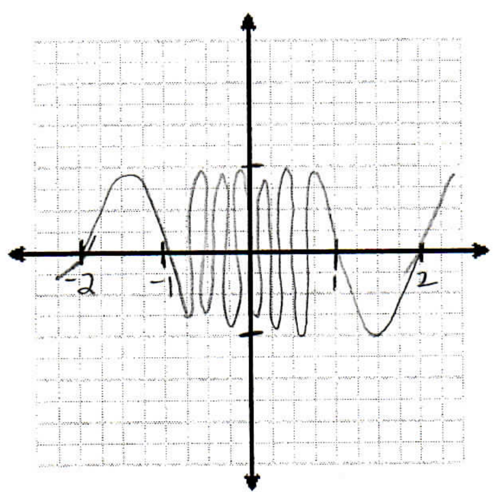
a. $\lim_{x\to 2} g(x)$ **if** $g(x)=\begin{cases} -\frac{1}{2}x+3, & x<2 \\ 3x-1, & x\geq 2 \end{cases}$ red green

$\lim_{x\to 2} g(x) = \text{DNE}$ b/c $\lim_{x\to 2^-} g(x) \neq \lim_{x\to 2^+} g(x)$

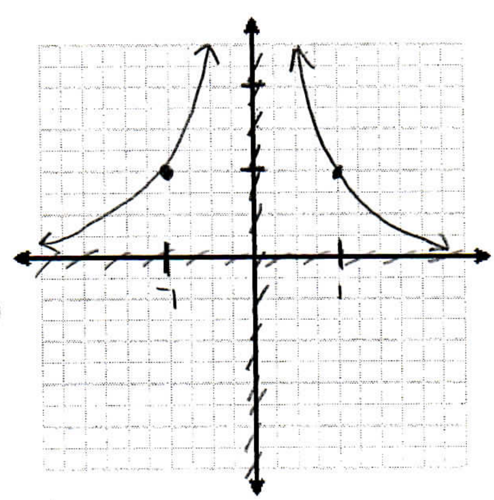


b. $\lim_{x\to 0} \sin\left(\frac{2\pi}{x}\right)$

zoom in at $x=0\ldots$ (2 or 3 times)




c. $\lim_{x\to 0} \frac{1}{x^4}$ since both the left & right side limits approach $\infty$, the limit does not exist (DNE)




11. Complete this statement: A limit does not exist if...

A. Left and Right limits exist but are not equal

B. a function oscillates between values as $x\to c$

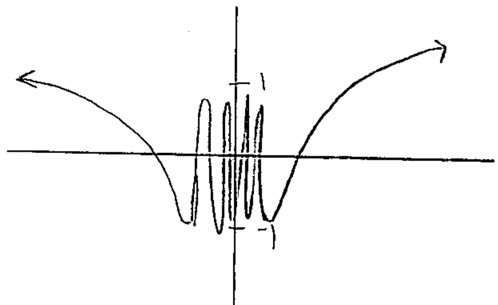
C. a function approaches $\pm\infty$ as $x\to c$

2.2 Limits that Involve Infinity p. 76 # 1–5, 13–16, 27, 53

Include a sketch for 53 to visualize the limits.

#1–5 Find a) $\lim_{x\to\infty} f(x)$ b.) $\lim_{x\to-\infty} f(x)$ c.) all HA

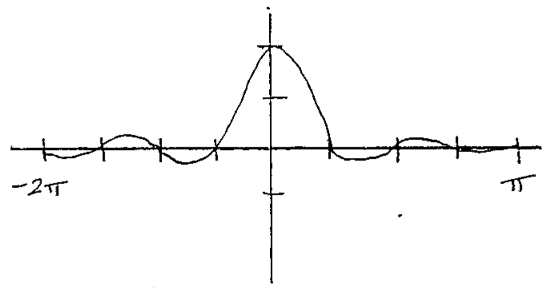
1.) $f(x) = \cos\left(\frac{1}{x}\right)$




a.) $\lim_{x\to-\infty} f(x) = 1$ b.) $\lim_{x\to\infty} f(x) = 1$

c.) HA @ $y = 1$

2. $f(x) = \frac{\sin 2x}{x}$




a.) $\lim_{x\to-\infty} f(x) = 0$ b.) $\lim_{x\to\infty} f(x) = 0$

c.) HA @ $y = 0$

3. $f(x) = \frac{e^{-x}}{x}$

a. $\lim_{x\to-\infty} f(x) = -\infty$

b. $\lim_{x\to\infty} f(x) = 0$

c.) $y = 0$

4. $f(x) = \frac{3x^3 - x + 1}{x + 3}$

a.) $\lim_{x\to\infty} f(x) = \infty$

b.) $\lim_{x\to-\infty} f(x) = -\infty$

c.) HA = none

5. $f(x) = \frac{3x+1}{|x| + 2}$

a. $\lim_{x\to\infty} f(x) = 3$

b. $\lim_{x\to-\infty} f(x) = -3$

c. HA @ $y = 3$ and $y = -3$

13. $\lim_{x\to 2^+} \frac{1}{x-2} = \infty$

14. $\lim_{x\to 2^-} \frac{x}{x-2} = -\infty$

15. $\lim_{x\to -3^-} \frac{1}{x+3} = -\infty$

16. $\lim_{x\to -3^+} \frac{x}{x+3} = -\infty$

a.) Find VA b.) Describe the behavior of f(x) to the left and right of VA

27. $f(x) = \dfrac{1}{x^2-4}$

$x^2-4=0$

$x^2=4$

VA $\boxed{x=\pm 2}$

$\lim_{x\to -2^-} f(x) = \infty$ $\lim_{x\to 2^-} f(x) = -\infty$

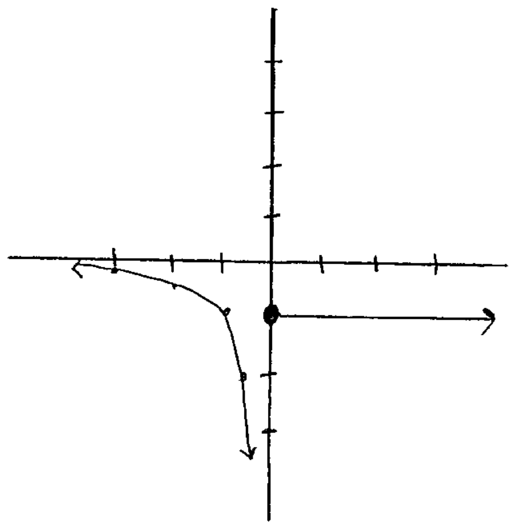
$\lim_{x\to -2^+} f(x) = -\infty$ $\lim_{x\to 2^+} f(x) = \infty$

53. Find Limit of f(x) as

a.) $x\to -\infty$ b.) $x\to\infty$ c.) $x\to 0^-$

d.) $x\to 0^+$

$$f(x) = \begin{cases} \dfrac{1}{x}, & x<0 \\ -1, & x\geq 0 \end{cases}$$



a.) $\lim_{x\to -\infty} f(x) = 0$ b.) $\lim_{x\to\infty} f(x) = -1$

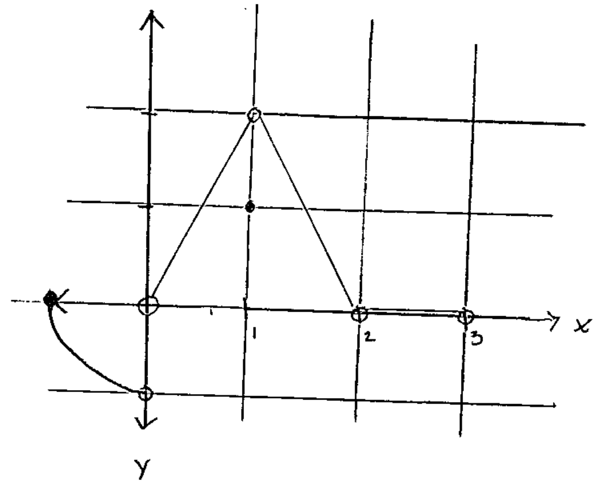
c.) $\lim_{x\to 0^-} f(x) = -\infty$ d.) $\lim_{x\to 0^+} f(x) = -1$

2.3A Continuity: The Calculus Definition p. 84 # 11–18, 41, 42

$x = c \qquad \lim_{x \to c} f(x) = f(c)$

"when you approach from the left and right of c, you will approach the same value as f(c)"

$$f(x) = \begin{cases} x^2 - 1, & -1 \le x < 0 \\ 2x & 0 < x < 1 \\ 1 & x = 1 \\ -2x + 4 & 1 < x < 2 \\ 0 & 2 < x < 3 \end{cases}$$




11.) (a) Does $f(-1)$ exist? yes
$f(-1) = 0$

(b.) Does $\lim_{x \to -1^+} f(x)$ exist? yes

(c.) Does $\lim_{x \to -1^+} f(x) = f(-1)$?
yes

(d.) Is $f(x)$ Continuous @ $x = -1$?
yes

12. (a) Does $f(1)$ exist? Yes
$f(1) = 1$

(b.) Does $\lim_{x \to 1} f(x)$ exist? yes

(c) Does $\lim_{x \to 1} f(x) = f(1)$? NO

(d.) Is $f$ continuous @ $x = 1$? NO

13. (a.) Is $f$ defined at $x = 2$?
NO since $\lim_{x \to 2} f(x) \neq f(2)$

(b) Is $f$ continvous @ $x = 2$? NO

14. At what values of $x$ is $f$ continuous?
Everywhere is $[-1, 3)$ except for $x = 0, 1, 2$

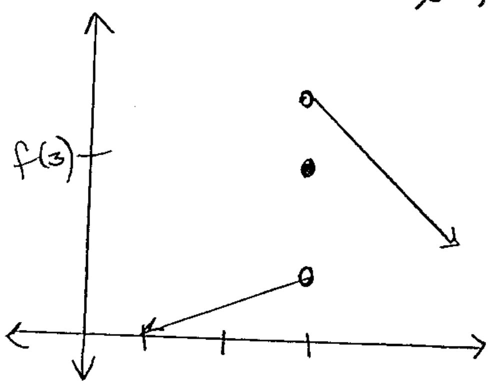
15. What value should be assigned to $f(2)$ to make the extended function Continuous at $x = 2$?
$\boxed{0}$

16. What new value should be assigned to $f(1)$ to make the new function Continuous @ $x = 1$
$\boxed{2}$

17. It is not possible because the limits from the left and right of $x = 0$ is different.

18. @ $x = 3$
let $f(3) = 0$ for it to be continuous.

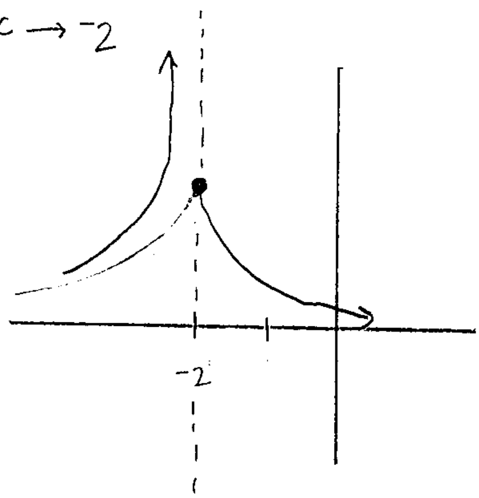
41. $f(3)$ exists but $\lim_{x \to 3} f(x)$ does not




42. $f(-2)$ exists,

$\lim_{x \to -2^+} f(x) = f(-2)$ but

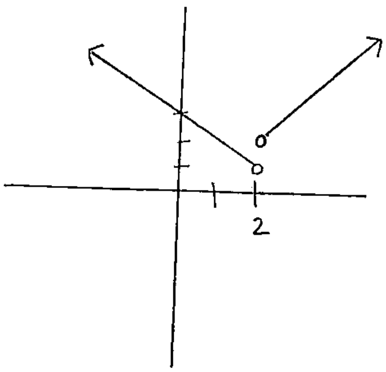
$\lim_{x \to -2} f(x) = \text{DNE}$



2.3B p.84 #19 - 22, $\boxed{45 - 50}$ ↳ 2.3C

$\boxed{23, 24}$ on back

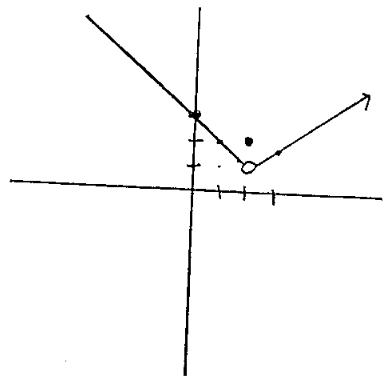
19.
$$f(x) = \begin{cases} 3 - x, & x < 2 \\ \frac{x}{2} + 1 & x > 2 \end{cases}$$




a.) discontinuous @ $x = 2$

b.) non-removable, L & R side limits ≠

20.
$$f(x) = \begin{cases} 3 - x, & x < 2 \\ 2, & x = 2 \\ x/2, & x > 2 \end{cases}$$

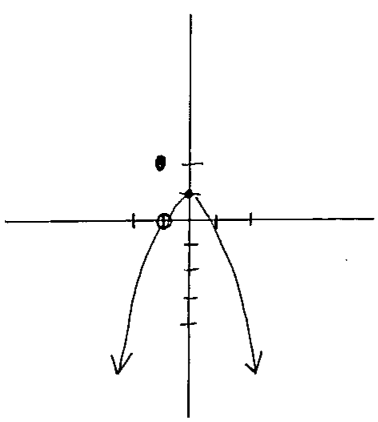


a.) discontinuous @ $x = 2$

b.) removable, L & R side limits are equal let $f(2) = 1$

21. (see notes)

22.
$$f(x) = \begin{cases} 1 - x^2, & x \neq -1 \\ 2, & x = -1 \end{cases}$$



a.) discontinuous @ $x = -1$

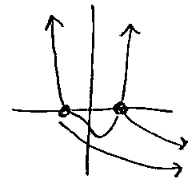
b.) removable, L & R side limits are equal, let $f(-1) = 0$

45. Is any real number exactly 1 less than its fourth power?

$$x^4 - 1 = x$$

$$x^2 - x - 1 = 0$$

Graph on calc and find zeros



$x \approx -.724$

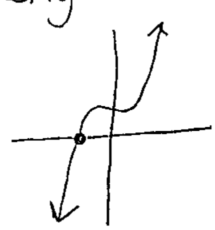
$x \approx 1.221$

46. Is any real number exactly 2 more than its cube?

$$x = 2 + x^3$$

$$0 = x^3 - x + 2$$

$$x \approx -1.521$$



47. Find a value for $a$ so that the function is continuous.

$$f(x) = \begin{cases} x^2-1, & x<3 \\ 2ax & x \geq 3 \end{cases}$$

This means, if I input a 3 in both the top and bottom equation, it should yield the same output.

$$(3)^2 - 1 = 2a(3)$$

$$\frac{8}{6} = \frac{6a}{6}$$

$$\boxed{a = \frac{4}{3}}$$

48. (same as 47)

$$f(x) = \begin{cases} 2x+3, & x \leq 2 \\ ax+1, & x>2 \end{cases}$$

$$2(2)+3 = a(2)+1$$

$$7 = 2a+1$$

$$6 = 2a$$

$$\boxed{a=3}$$

49. $f(x) = \begin{cases} 4-x^2, & x<-1 \\ ax^2-1, & x \geq -1 \end{cases}$

$$4-(-1)^2 = a(-1)^2 - 1$$

$$3 = a-1$$

$$\boxed{a=4}$$

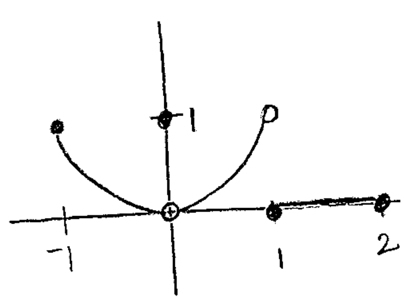
50. $f(x) = \begin{cases} x^2+x+a, & x<1 \\ x^3, & x \geq 1 \end{cases}$

$$(1)^2 + (1) + a = (1)^3$$

$$2 + a = 1$$

$$\boxed{a=-1}$$

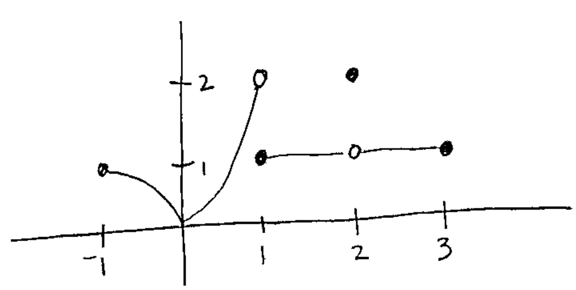
23.




a.) discontinuous @ $x=0, 1$

b.) removable at $x=0$
L & R Limits =
let $f(0)=0$
not removable at $x=1$
L & R Limits ≠

24.

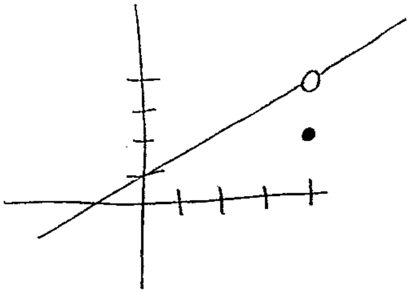



a.) discontinuous @ $x=1, 2$

b.) not removable at $x=1$, L & R limits are ≠
removable @ $x=2$, let $f(2)=1$

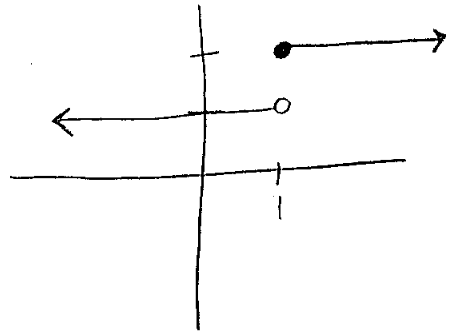
2.3C The Intermediate Value Theorem p. 85 # $\underbrace{45\text{-}50}_{\text{on 2.3B hw}}$, 43, 44, 56, 59

43. $f(4)$ exists, $\lim_{x\to 4} f(x)$ exists, but $f$ is not continuous at $x=4$



* answers may vary

44. $f(x)$ is continuous for all $x$ except $x=1$, where $f$ has a non removable discontinuity



* answers may vary

56. On which interval is $f(x) = \frac{1}{\sqrt{x}}$ not continuous?

$x > 0$

[B] $[0, \infty)$

↑

cannot include zero

59. $$f(x) = \frac{\cancel{x}\cancel{(x-1)}(x-2)^{\cancel{2}\,1}\cancel{(x+1)^2}(x-3)^2}{\cancel{x}\cancel{(x-1)}\cancel{(x-2)}\cancel{(x+1)^2}(x-3)^{\cancel{3}}}$$

not removable @ $x=3$

E

2.4A Average Rates of Change p. 92 # 1-6, 7* for 7, estimate coordinates for points Q-$Q_4$

Find the A.R.O.C. of the function over each interval

1. $f(x) = x^3+1$

(a.) [2,3]

$$\frac{f(3)-f(2)}{3-2} = \frac{(3^3+1)-(2^3+1)}{1} = 28-9 = 19$$

(b.) [-1,1]

$$\frac{f(1)-f(-1)}{1-(-1)} = \frac{(1^3+1)-((-1)^3+1)}{2} = \frac{2-(0)}{2} = 1$$

2. $f(x) = \sqrt{4x+1}$

(a) [0,2]

$$\frac{f(2)-f(0)}{2-0} = \frac{(\sqrt{4(2)+1})-(\sqrt{4(0)+1})}{2} = \frac{\sqrt{9}-\sqrt{1}}{2} = \frac{3-1}{2} = 1$$

(b) [10,12]

$$\frac{f(12)-f(10)}{12-10} = \frac{(\sqrt{4(12)+1})-(\sqrt{4(10)+1})}{2} = \frac{\sqrt{49}-\sqrt{41}}{2} \approx 0.298$$

3. $f(x) = e^x$

(a) [-2,0]

$$\frac{f(0)-f(-2)}{0--2} = \frac{e^0-e^{-2}}{2} = \frac{1-e^{-2}}{2} \approx 0.432$$

(b.) [1, 3]

$$\frac{f(3)-f(1)}{3-1} = \frac{e^3-e^1}{2} \approx 8.683$$

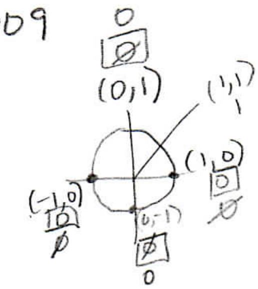
4. $f(x) = \ln x$

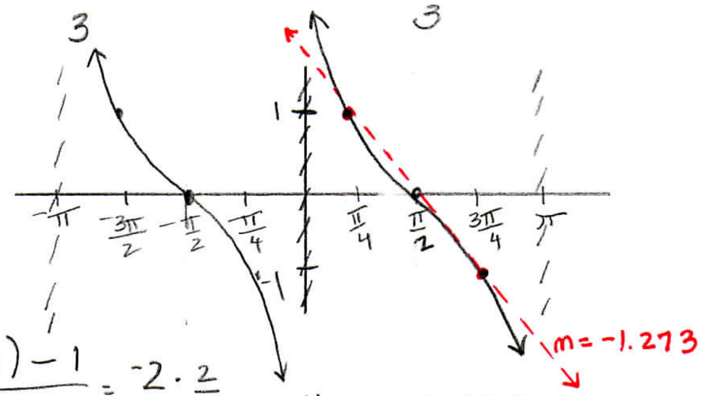
(a.) $[1,4]$

$$\frac{f(4)-f(1)}{4-1} = \frac{\ln 4 - \cancel{\ln 1}^{\,0}}{3} \approx 0.462$$

(b.) $[100,103]$

$$\frac{f(103)-f(100)}{103-100} = \frac{\ln 103 - \ln 100}{3} = \frac{\ln\left(\frac{103}{100}\right)}{3} \approx 0.009$$

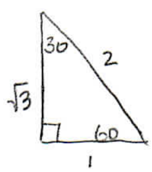





5. $f(x) = \cot x$

(a.) $[\pi/4, \frac{3\pi}{4}]$

$$\frac{f(\frac{3\pi}{4}) - f(\frac{\pi}{4})}{\frac{3\pi}{4} - \frac{\pi}{4}} = \frac{(-1)-1}{\frac{\pi}{2}} = -2 \cdot \frac{2}{\pi} = \frac{-4}{\pi} \approx -1.273$$




$\frac{\pi}{6} = 30°$ $\quad \cot\theta = \frac{adj}{opp}$ $\quad \cot 30 = \sqrt{3}$

(b.) $[\pi/6, \pi/2]$

$$\frac{f(\frac{\pi}{2}) - f(\frac{\pi}{6})}{\frac{\pi}{2} - \frac{\pi}{6}} = \frac{0-\sqrt{3}}{\frac{\pi}{3}} = -\sqrt{3}\cdot\frac{3}{\pi} \approx -1.653$$

6. $f(x) = 2 + \cos t$

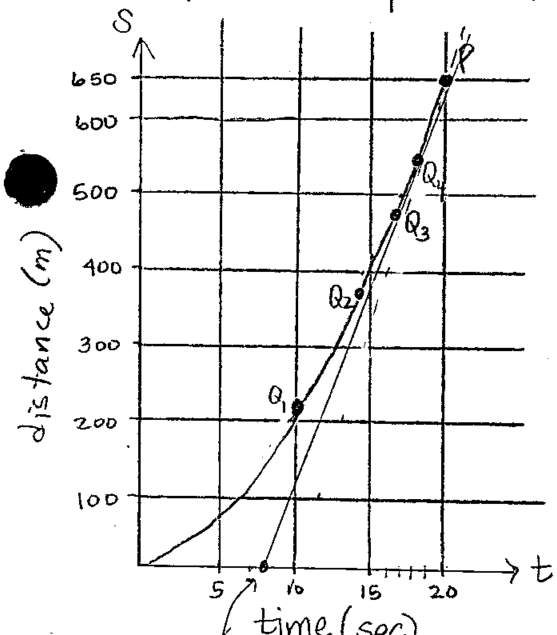
(a.) $[0,\pi]$

$$\frac{f(\pi)-f(0)}{\pi - 0} = \frac{(2+\cos\pi)-(2+\cos(0))}{\pi} = \frac{(2+-1)-(2+1)}{\pi} = \frac{1-3}{\pi} = \frac{-2}{\pi} \approx -0.636$$

(b.) $[-\pi,\pi]$

$$\frac{f(\pi)-f(-\pi)}{\pi-(-\pi)} = \frac{(2+\cos\pi)-(2+\cos(-\pi))}{2\pi} = \frac{1-(2+-1)}{2\pi} = \frac{0}{2\pi} = 0$$

2.4A cont. p.92 #7




≈ tangent line x-int. (7,0)

a.) Estimate the slopes of the secants $PQ_1$, $PQ_2$, $PQ_3$ and $PQ_4$ (create a table)

$PQ_1 = \overset{Q_1}{(10, 220)}\ \overset{P}{(20, 650)}$

$$\frac{650-220}{20-10} = 43$$

$PQ_2 = \overset{Q_2}{(14, 375)}\ \overset{P}{(20, 650)}$

$$\frac{650-375}{20-14} = 46$$

$PQ_3 = \overset{Q_3}{(16, 475)}\ \overset{P}{(20, 650)}$

$$\frac{650-475}{20-16} = 58$$

$PQ_4 = \overset{Q_4}{(18, 550)}\ \overset{P}{(20, 650)}$

$$\frac{650-550}{20-18} = \frac{100}{2} = 50$$

| secant | slope |
|---|---|
| $PQ_1$ | 43 |
| $PQ_2$ | 43 |
| $PQ_3$ | 44 |
| $PQ_4$ | 50 |

unit = meters per second = m/s

b.) Estimating the tangent line

(7,0) (20,650)

$$\frac{650}{20-7} = \frac{650}{13} = 50$$

2.4B The Slope of a Curve: Tangent and Normal Lines EQ

p. 92 # 9, 10, 12, 8*, 38 (for 8, estimate points)

9. $y = x^2$ at $x = -2$

$f(-2) = (-2)^2$

$f(-2) = 4 \quad (-2, 4)$

$f(-2) \quad f(-2+h)$

a.) $\lim_{h \to 0} \dfrac{f(-2+h) - f(-2)}{h} = \dfrac{\overset{(-2+h)(-2+h)}{(-2+h)^2} - 4}{h} = \dfrac{\cancel{4} - 4h + h^2 - \cancel{4}}{h} = \dfrac{\cancel{h}(-4+h)}{\cancel{h}} = -4 = m$

b. $m = -4 \quad (-2, 4)$

$\boxed{y - 4 = -4(x + (+2))}$

$y - 4 = -4x - 8$

$+4 \qquad +4$

$y = -4x - 4$

c. Normal Line

$m = \frac{1}{4}$

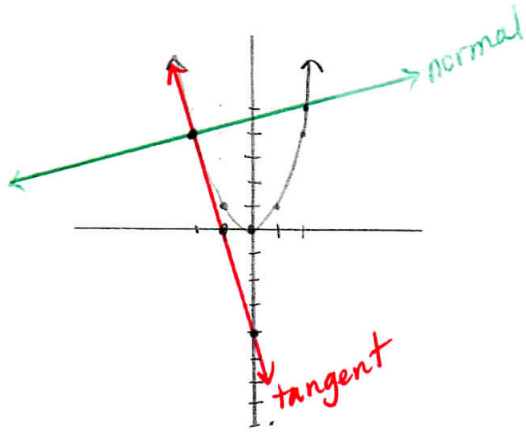
$\boxed{y - 4 = \frac{1}{4}(x + 2)}$

$y - 4 = \frac{1}{4}x + \frac{1}{2}$

$+4 \qquad +\frac{8}{2}$

$y = \frac{1}{4}x + \frac{9}{2}$

d.




$x(x-4)$

10. $y = x^2 - 4x$ @ $x = 1$

$f(1) = (1)^2 - 4(1) \quad (1, -3)$

$= 1 - 4$

$= -3$

$f(1+h) = (1+h)^2 - 4(1+h)$

$(1+h)(1+h) - 4 - 4h$

$1 + 2\cancel{h} + h^2 - 4 - \cancel{4}h$

$h^2 - 2h - 3$

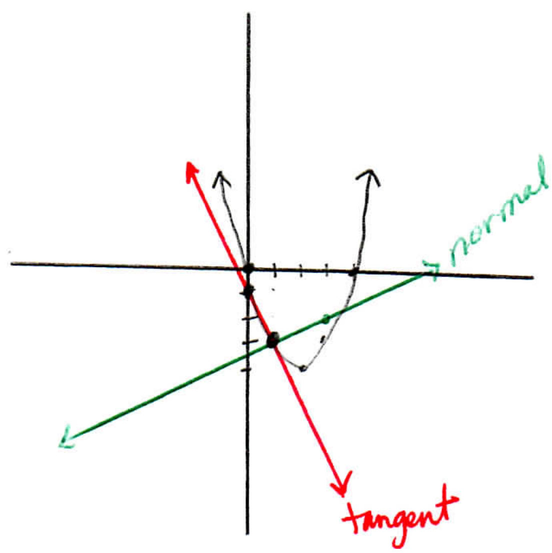
a) $\lim_{h \to 0} \dfrac{f(1+h) - f(1)}{h} = \dfrac{h^2 - 2h - \cancel{3} + \cancel{3}}{h} = h - 2 = -2 = m$

b) $m = -2 \quad (1, -3)$

$\boxed{y + 3 = -2(x - 1)}$

c.) $m = \frac{1}{2} \quad (1, -3)$

$\boxed{y + 3 = \frac{1}{2}(x - 1)}$



12. $y = x^2 - 3x - 1$ @ $x = 0$ $\quad f(h+0) = h^2 - 3h - 1$

$f(0) = -1$

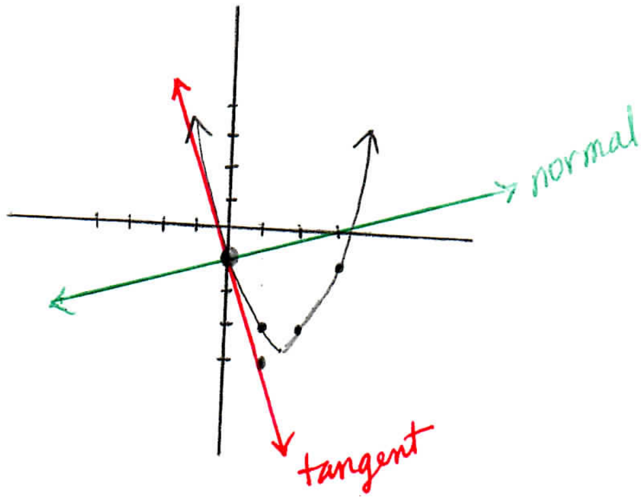
a) $\lim\limits_{h \to 0} \dfrac{f(h+0) - f(0)}{h} = \dfrac{h^2 - 3h - \cancel{1} + \cancel{1}}{h} = \dfrac{\cancel{h}(h-3)}{\cancel{h}} = h - 3 = -3 = m$

b.) $m = -3 \quad (0,-1)$

$y + 1 = -3(x - 0)$

c.) $m = \frac{1}{3} \quad (0,-1)$

$y + 1 = \frac{1}{3}(x - 0)$




8. $P(10,80)$ $Q_1(5,20)$ $Q_2(7,38)$ $Q_3(8.5,56)$ $Q_4(9.5,72)$

a.)

| secant | slope |
|---|---|
| $PQ_1$ | 12 |
| $PQ_2$ | 14 |
| $PQ_3$ | 16 |
| $PQ_4$ | 16 |

$PQ_1 = \dfrac{80-20}{10-5} = 12$ $\quad PQ_2 = \dfrac{80-38}{10-7} = 14$

$PQ_3 = \dfrac{80-56}{10-8.5} = 16$ $\quad PQ_4 = \dfrac{80-72}{10-9.5} = 16$

b.) $\approx 16$ m/s (estimate)

38. $f(x) = x^2 + x \quad [1,3]$

$\dfrac{f(3) - f(1)}{3-1} = \dfrac{(3^2+3) - ((1)^2+1)}{2} = \dfrac{12-2}{2} = 5$ (E)

Chapter 2 Review p.95 #1-3, 6, 7, 15-27, 39, 41, 47

1.) $\lim_{x \to -2} x^3 - 2x^2 + 1$

$(-2)^3 - 2(-2)^2 + 1$

$-8 - 2(4) + 1$

$-8 - 8 + 1$

$-16 + 1$

$-15$

2. $\lim_{x \to -2} \dfrac{x^2+1}{3x^2-2x+5}$

$\dfrac{(-2)^2+1}{3(-2)^2-2(-2)+5}$

$\dfrac{4+1}{3(4)+(4)+5} = \dfrac{5}{21}$

3. $\lim_{x \to 4} \sqrt{1-2x}$

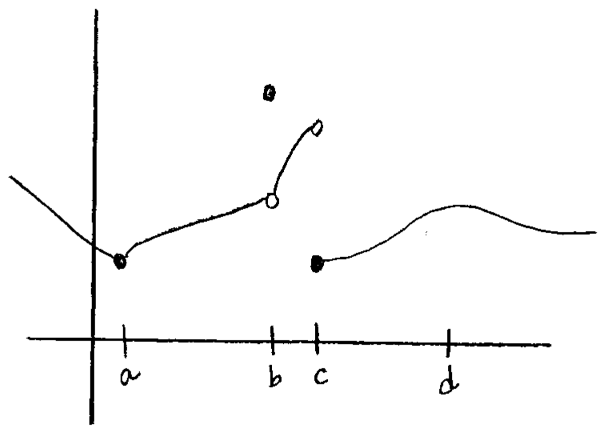
$\sqrt{1-2(4)}$

$\sqrt{1-8}$

$\sqrt{-7}$ DNE

6. $\lim_{x \to \pm\infty} \dfrac{2x^2+3}{5x^2+7} = \dfrac{2}{5}$

7. $\lim_{x \to \pm\infty} \dfrac{x^4+x^3}{12x^3+128} = \text{DNE}$




15. $\lim_{x \to d} f(x) =$ yes

16. $\lim_{x \to c^+} f(x) =$ yes

17. $\lim_{x \to c^-} f(x) =$ yes

18 $\lim_{x \to c} f(x) =$ DNE

19. $\lim_{x \to b} f(x) =$ yes

20. $\lim_{x \to a} f(x) =$ yes

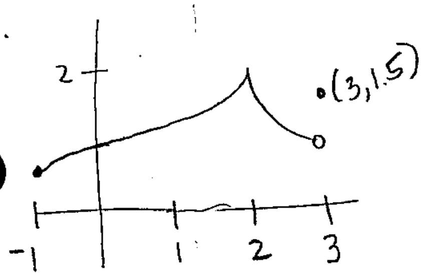
21. yes (continuous @ a)

22. no removable discontinuity @ b

23. no non-removable discontinuity @ c

24. yes, Continuous @ d.

25.




a) $\lim_{x \to 3^-} g(x) = 1$

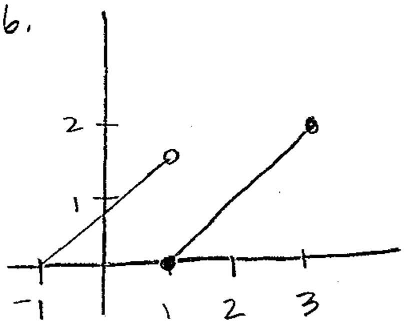
b) $g(3) = 1.5$

c.) discontinuous @ $x = 3$

d.) removable @ $x = 3$

e.) by letting $f(3) = 1$

26.




a.) $\lim_{x \to 1^-} k(x) = 1.5$

b.) $\lim_{x \to 1^+} k(x) = 0$

c.) $k(1) = 0$

d.) $k(x)$ is not continuous at $x=1$

e.) discontinuous @ $x=1$

(f) not removable @ $x=1$ due to jump

27. $f(x) = \dfrac{x+3}{x+2}$

VA @ $x=-2$

HA @ $y=1$

$\lim_{x \to -\infty} f(x) = 1$

$\lim_{x \to \infty} f(x) = 1$

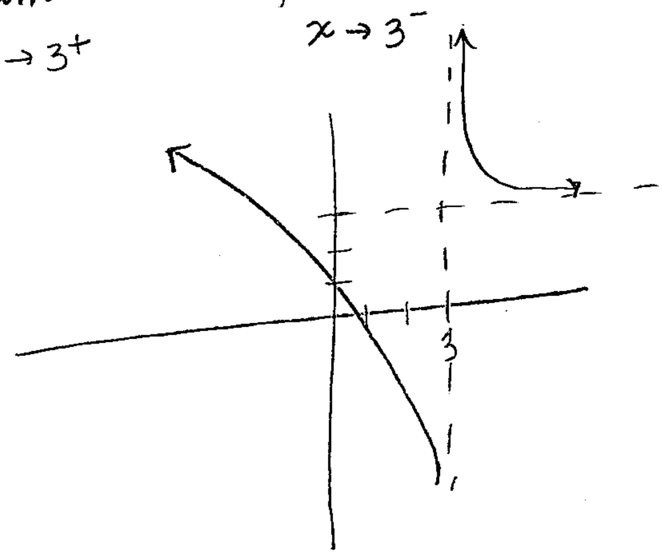
39. $f(x) = \begin{cases} \dfrac{x^2+2x-15}{x-3}, & x \neq 3 \\ k & ; x = 3 \end{cases}$ DNE @ $x=3$

$\dfrac{(x+5)\cancel{(x-3)}}{\cancel{(x-3)}}$ $3+5 = 8 = \lim$

$k=8$

41. $\lim_{x \to \infty} f(x) = 3$, $\lim_{x \to -\infty} f(x) = \infty$

$\lim_{x \to 3^+} f(x) = \infty$, $\lim_{x \to 3^-} f(x) = -\infty$




47. $f(x) = x^2 - 3x$ $P = (1, f(1))$

$f(1) = (1)^2 - 3(1)$ $(1,-2)$

$= 1-3$

$= -2$

$f(h+1) = (h+1)^2 - 3(h+1)$

$= h^2 + 2h + 1 - 3h - 3$

$\lim_{h \to 0} \dfrac{(h^2 - h \cancel{-2}) + (\cancel{+2})}{h}$

$\downarrow$ $\dfrac{\cancel{h}(h-1)}{\cancel{h}} = \boxed{-1}$

b.) $y+2 = -1(x-1)$

c.) $y+2 = 1(x-1)$